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Stability of a Class of Switched Stochastic Nonlinear Systems under Asynchronous Switching

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Abstract: The stability of a class of switched stochastic nonlinear retarded systems with asynchronous switching controller is investigated. By constructing a virtual switching signal and using the average dwell time approach incorporated with Razumikhin-type theorem, the sufficient criteria for pth moment exponential stability and global asymptotic stability in probability are given. It is shown that the stability of the asynchronous stochastic systems can be guaranteed provided that the average dwell time is sufficiently large and the mismatched time between the controller and the systems is sufficiently small. This result is then applied to a class of switched stochastic nonlinear delay systems where the controller is designed with both state and switching delays. A numerical example illustrates the effectiveness of the obtained results.

Keywords: Asynchronous switching, average dwell time, Razumikhin-type theorem, stochastic stability, switched stochastic nonlinear systems, time-delays.

1. INTRODUCTION

Switched systems are a special class of hybrid systems, in the sense that the former is described by a family of finite subsystems whose active modes are governed by a switching rule. Switched systems are reasonable models for various practical systems, such as networked control systems, communication systems, flight control, etc. [1-3]. One is usually interested in the stability, controllability, robustness, passivity, optimal control, sliding mode control, etc., of such systems [4-14], among which the stability is the most concerned and a number of tools, e.g., multiple Lyapunov functions, average dwell time approach, have been proposed from various perspectives [11,15-17]. On the other hand, time delays are often seen in practical engineering systems, which can be a severe factor that deteriorates the system performance. A large volume of studies can be seen from the literature [18-20]; some are undertaken within the switched system framework [21-27].

Two types of controllers, the mode-dependent and the mode-independent, are seen for switched systems. It is believed that the mode-dependent controller is less conservative as it takes advantage of more information of the system. The mode-dependent controller is often assumed to be ideally synchronous with the switching of systems [25] which, however, may not be true in reality due to the presence of time delays. Specifically, on the one hand, time-delay often appears in switched systems either in input control or in output measurements. The former is mainly due to actuator dynamics, the calculation of the control gains, the communication delay between the controller and the actuator, etc. while the latter can be caused by the communication delay between the sensor and the controller, etc. In some cases delay can indirectly be induced by a phase lag in filtering out the noise from the measurements. On the other hand, in the practical implementation, due to unknown abrupt phenomena such as component and interconnection failures, detecting the switching rules also takes time. Those thus present a great challenge at the boundary of switched systems and time delay systems. Then the so-called asynchronous switching is proposed, and a number of efforts have been made, for example, the admissible delay of asynchronous switching are given in [28,29]; state feedback stabilization, input-to-state stabilization and output feedback stabilization are studied in [30-36]; results have also been reported for Markov jump linear systems [37-39], just to name a few.

Switched stochastic systems have recently been popular due to the significant role played by the stochastic modelling in many branches of engineering disciplines. Consequently, the study of control synthesis for such systems with asynchronous switching, e.g., robust stabilization, H_∞ filtering, have been widely seen [40-43].

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Using the boundedness assumption of nonlinear term, a linear matrix inequality (LMI) approach incorporated with Lyapunov method is developed to meet the goal. In another line switched stochastic retarded systems have received much attention in recent years. Such systems consist of a set of stochastic retarded subsystems that are described by stochastic functional differential equations. Works in related areas can be found in, for example, [44] for the pth moment input-to-state stability of stochastic nonlinear retarded systems under Markovian switching, [24] for the stability under average dwell time switching signal, and so forth. Despite all these results, to date switched stochastic nonlinear retarded systems under asynchronous switching have received little attention, which motivates this study for us.

In this paper, we investigate the stability of a class of switched stochastic nonlinear retarded systems under asynchronous switching. The main challenge for a mode dependent controller design is to deal with the mismatched period due to the existence of detection delays. Our efforts are made towards the stability criteria for such systems with respect to the mismatched interval. For a more realistic situation, we consider the controller with both state and switching delays. To describe and deal with the asynchronous switching phenomenon, a virtual switching signal is constructed and applied. Finally, a sufficient Razumikhin-type stability criterion is derived to guarantee the stability of the closed-loop system under average dwell time approach.

The remainder of the paper is organized as follows. The problem is formulated and necessary definitions are given in Section 2. The main results are then discussed in Section 3, with an application given in Section 4. Section 5 concludes the paper.

Notions: $\mathbb{N}_+$ and $\mathbb{R}_+$ denote the set of positive integer and nonnegative real numbers, respectively. Let $\mathbb{N} = \mathbb{N}_+ \cup \{0\}$. If x and y are real numbers, then $x \wedge y$ denotes the minimum of x and y. For vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm. Let $\tau \geq 0$ and $C([-\tau,0];\mathbb{R}^n)$ denote the family of all continuous $\mathbb{R}^n$-valued functions φ on $[-\tau, 0]$ with the norm $\|\varphi\| = sup\{|\varphi(\theta)| : -\tau \leq \theta \leq 0\}$. Let $C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau,0];\mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$. For $t \geq 0$, let $L^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$ denote the family of all $\mathcal{F}_t$-measurable $C([-\tau,0];\mathbb{R}^n)$-valued random variables $\phi = \{\phi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0}\mathbb{E}\{|\phi(\theta)|^p\} < \infty$. The transpose of vectors and matrices is denoted by superscript T. C^i denotes all the ith continuous differential functions; $C^{i,k}$ denotes all the functions with ith continuously differentiable first component and kth continuously differentiable second component. Finally, the composition of two functions $\alpha : A \to B$ and $\beta : B \to C$ is denoted by $\alpha \circ \beta : A \to C$.

2. PRELIMINARIES

Consider the following switched stochastic nonlinear retarded systems

$$dx = f_{\sigma(t)}(t,x,x_t,u(t))dt + g_{\sigma(t)}(t,x,x_t,u(t))dw, \qquad (1)$$

where $x = x(t) \in \mathbb{R}^n$ is the state vector, $x_t = \{x(t+\theta) : -\tau \leq \theta \leq 0\}$ is $C([-\tau,0];\mathbb{R}^n)$-valued random process, $u(t) \in \mathcal{L}^l_\infty$ is the control input. $\mathcal{L}^l_\infty$ denotes denotes the set of all the measurable and locally essentially bounded input $u(t) \in \mathbb{R}^l$ on $[t_0,\infty)$ with the norm

$$\|u(s)\|_{[t_0,\infty)} = \sup_{s\in[t_0,\infty)} \inf_{\mathcal{A}\in\Omega,\mathbb{P}(\mathcal{A})=0} \sup\{u(w,s) : w \in \Omega \setminus \mathcal{A}\} \qquad (2)$$

$w(t)$ is the m-dimensional Brownian motion defined on the complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq t_0},\mathbb{P})$, with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t\geq t_0}$ being a filtration and $\mathbb{P}$ being a probability measure. $\sigma : [t_0,\infty) \to \mathcal{S}$ ($\mathcal{S}$ is the index set, and may be infinite) is the switching law and is right hand continuous and piecewise constant on t. $\sigma(t)$ discussed in this paper is time dependent, and the corresponding switching times are $t_1 < t_2 < \cdots < t_l < \cdots$. The i_lth subsystems will be activated at time interval $[t_l, t_{l+1})$. Specially, when $t = t_0$ (t_0 is the initial time), suppose $\sigma_0 = \sigma(t_0) = i_0 \in \mathcal{S}$. For all $i \in \mathcal{S}$, $f_i : \mathbb{R}_+ \times \mathbb{R}^n \times C([-\tau,0];\mathbb{R}^n) \times \mathbb{R}^l \to \mathbb{R}^n$ and $g_i : \mathbb{R}_+ \times \mathbb{R}^n \times C([-\tau,0];\mathbb{R}^n) \times \mathbb{R}^l \to \mathbb{R}^{n\times m}$ are continuous with respect to t, x, u, and satisfy uniformly locally Lipschitz condition with respect to x, u, and $f_i(t,0,0,0) \equiv 0$, $g_i(t,0,0,0) \equiv 0$.

In practice, instantaneous switching signal detection is impossible. In this paper, we are concerned the stability of systems under the following mode-dependent state feedback law: $u(t) = h_{\sigma'(t)}(t,x_t)$, on $t \geq t_0$ with initial data $x_0 = \{x(t_0+\theta) : -\tau \leq \theta \leq 0\} = \xi \in C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ and $r_0 = r(t_0+\theta) = r(t_0) = i_0$. $\sigma'(t) = \sigma(t - d_\sigma(t))$ is utilized to denote the practical switching signal of controller, where $0 \leq d_\sigma(t) \leq \bar{d}$. Further, we assume that, the detected switching signal $\sigma'(t)$ is causal, i.e., the ordering of the switching times of $\sigma'(t)$ is the same as the ordering of the corresponding switching times of $\sigma(t)$. Further, we also assume that $\bar{d} \leq \inf_{l\in\mathbb{N}}\{t_{l+1} - t_l\}$, which guarantees that there always exists a period in which the controller and the system operate synchronously. This period is called matched period. Let $\{t_l'\}_{l\geq 1}$ denotes the switching times of $\sigma'(t)$, $t_l' = t_l + d_{\sigma(t_l)}(t_l)$, then $\sigma'(t_l') = \sigma(t_l) = i_l$, and $t_1 < t_1' < t_2 < t_2' < \cdots < t_l < t_l' < t_{l+1} < \cdots$. We assume $t_0' = t_0$.

Due to the existence of the detection delay, there exists a period in $[t_l, t_{l+1})$ such that the mode-dependent feedback control input $u(t)$ and the i_lth subsystem operate asynchronously. We call the time interval $[t_l, t_l')$ the mismatched period, and $[t_l', t_{l+1})$ the matched period. For convenience, let $T_a(t_l,t_{l+1}) = [t_l,t_l')$, $T_s(t_l,t_{l+1}) = [t_l', t_{l+1})$. For any $s \geq t_0$, let $T_a(t-s)$ denote the total time of the mismatched time interval on $[s,t]$. Then, for any $l \in \mathbb{N}_+$, we have

$$T_a(t-t_l)=\begin{cases} t-t_l & t\in T_a(t_l,t_{l+1}) \\ T_a(t_{l+1}-t_l) & t\in T_s(t_l,t_{l+1}) \\ T_a(t_{l+1}-t_l)+t-t_{l+1} & t\in T_a(t_{l+1},t_{l+2}) \\ \sum_{i=l}^{l+1} T_a(t_{i+1}-t_i) & t\in T_s(t_{l+1},t_{l+2}) \\ \cdots & \cdots \\ \sum_{i=l}^{n-1} T_a(t_{i+1}-t_i)+t-t_n & t\in T_a(t_n,t_{n+1}) \\ \sum_{i=l}^{n} T_a(t_{i+1}-t_i) & t\in T_s(t_n,t_{n+1}), \end{cases}$$

where $T_a(t_{l+1}-t_l)$ is the length of the $T_a(t_l,t_{l+1})$.

For each $i\in\mathcal{S}$, $h_i:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}^l$ is a smooth function. Then, the closed-loop system can be transformed into the following retarded-type systems

$$dx=\bar{f}_{\bar{\sigma}(t)}(t,x,x_t)dt+\bar{g}_{\bar{\sigma}(t)}(t,x,x_t)dw, \tag{3}$$

where $\bar{f}_{\bar{\sigma}(t)}(t,x,x_t)=f_{\sigma(t)}(t,x,x_t,h_{\sigma'(t)}(t,x_t))$, $\bar{g}_{\bar{\sigma}(t)}(t,x,x_t)=g_{\sigma(t)}(t,x,x_t,h_{\sigma'(t)}(t,x_t))$. $\bar{\sigma}(t)$ is a virtual switching signal from $[t_0,\infty)$ to $\mathcal{S}\times\mathcal{S}$ with $\bar{\sigma}(t)=(\sigma(t),\sigma'(t))$ and switching times $\{\bar{t}_l\}_{l\geq 0}$ $(\bar{t}_0=t_0)$. Clearly, we have $\bar{t}_{2l-1}=t_l$ and $\bar{t}_{2l}=t_l'$, for any $l\geq 1$. Assume that the composite functions $\bar{f}$ and $\bar{g}$ are sufficiently smooth, such that system (3) has an unique solution on $t\geq t_0-\tau$.

The following definitions are needed for the stability of closed-loop system (3).

Definition 1: The equilibrium $x(t)=0$ of system (3) is globally asymptotically stable in probability (GASiP), if for any $\varepsilon>0$, there exists a class $\mathcal{KL}$ function β such that $\mathbb{P}\{|x|<\beta(\mathbb{E}\{\|\xi\|\},t-t_0)\}\geq 1-\varepsilon,\forall t\geq t_0$.

Remark 1: Class $\mathcal{K}$, $\mathcal{K}_\infty$, $\mathcal{KL}$ functions are defined in [45]. And in the sequel, class $\mathcal{CK}_\infty$ and $\mathcal{VK}_\infty$ function are the two subsets of class $\mathcal{K}_\infty$ functions that are convex and concave, respectively.

Definition 2: The equilibrium $x(t)=0$ of system (3) is pth moment exponentially stable, if there exists a class $\mathcal{KL}$ function β (where $\beta(\cdot,t)$ will converge to zero by the way of exponential decay as $t\to\infty$), such that

$$\mathbb{E}\{|x|^p\}<\beta(\mathbb{E}\{\|\xi\|^p\},t-t_0),\quad \forall t\geq t_0. \tag{4}$$

Definition 3 [15]**:** For any given constants $\tau^*>0$ and N_0, let $N_\sigma(t,s)$ denote the switch number of $\sigma(t)$ in $[s,t)$, for any $t>s\geq t_0$, and let

$$S[\tau^*,N_0]=\{\sigma(\cdot):N_\sigma(t,s)\leq N_0+\frac{t-s}{\tau^*},\forall s\in[t_0,t)\}.$$

Then τ^* is called the average dwell-time of $S[\tau^*,N_0]$, and $\tau_\sigma\triangleq\sup_{t\geq t_0}\sup_{t>s\geq t_0}\frac{t-s}{N_\sigma(t,s)-N_0}$ is called the average dwell-time of $\sigma(\cdot)$.

Lemma 1 [23]**:** If $\sigma(\cdot)\in S[\tau^*,N_0]$, then $\sigma'(\cdot)\in S[\tau^*,N_0+\frac{\bar{d}}{\tau^*}]$ and $\bar{\sigma}(\cdot)\in S[\frac{\tau^*}{2},2N_0+\frac{\bar{d}}{\tau^*}]$.

Definition 4 [18]**:** For any given $V\in C^{2,1}(\mathbb{R}^n\times\mathbb{R}_+\times\mathcal{S}\times\mathcal{S};\mathbb{R}_+)$, define a diffusion operator associated with system (3), $\mathcal{L}V$, from $C([-\tau,0];\mathbb{R}^n)\times\mathbb{R}_+\times\mathcal{S}\times\mathcal{S}$ to $\mathbb{R}$, by

$$\begin{aligned}&\mathcal{L}V(x_t,t,\bar{\sigma}(t))\\&=\frac{\partial V(x,t,\bar{\sigma}(t))}{\partial t}+\frac{\partial V(x,t,\bar{\sigma}(t))}{\partial x}\bar{f}_{\bar{\sigma}(t)}(t,x,x_t)\\&+\frac{1}{2}trace[\bar{g}^T_{\bar{\sigma}(t)}(t,x,x_t)\frac{\partial^2V(x,t,\bar{\sigma}(t))}{\partial x^2}\bar{g}_{\bar{\sigma}(t)}(t,x,x_t)],\end{aligned}\tag{5}$$

where $V(\cdot,t,\bar{\sigma}(t))\triangleq V(\cdot,t,\sigma(t),\sigma'(t))$.

In what follows let $V_{\bar{\sigma}(t)}(\cdot,t)$ denote $V(\cdot,t,\bar{\sigma}(t))$.

3. MAIN RESULTS

Based on the average dwell-time approach, we give the sufficient criteria for GASiP and pth moment exponential stability for a class of switched stochastic nonlinear retarded systems. We first consider the stability of ordinary switched systems under asynchronous switching and then the case with retarded-type state-feedback delay.

Let $x_t=x(t)$ in (3), then, it can be transformed into

$$\mathrm{d}x=\bar{f}_{\bar{\sigma}(t)}(t,x)\mathrm{d}t+\bar{g}_{\bar{\sigma}(t)}(t,x)\mathrm{d}w, \tag{6}$$

where $\bar{f}_{\bar{\sigma}(t)}(t,x)\triangleq f_{\sigma(t)}(t,x,h_{\sigma'(t)}(t,x))$, $\bar{g}_{\bar{\sigma}(t)}(t,x)\triangleq g_{\sigma(t)}(t,x,h_{\sigma'(t)}(t,x))$. Similarly, for any $V\in C^{2,1}(\mathbb{R}^n\times\mathbb{R}_+\times\mathcal{S}\times\mathcal{S};\mathbb{R}_+)$, we can define the infinitesimal operator L from $\mathbb{R}^n\times\mathbb{R}_+\times\mathcal{S}\times\mathcal{S}$ to $\mathbb{R}$, associated with system (6), by

$$\begin{aligned}LV_{\bar{\sigma}(t)}(x,t)&=\frac{\partial V_{\bar{\sigma}(t)}(x,t)}{\partial t}+\frac{\partial V_{\bar{\sigma}(t)}(x,t)}{\partial x}\bar{f}_{\bar{\sigma}(t)}(t,x)\\&+\frac{1}{2}trace\times[\bar{g}^T_{\bar{\sigma}(t)}(t,x)\frac{\partial^2V_{\bar{\sigma}(t)}(x,t)}{\partial x^2}\bar{g}_{\bar{\sigma}(t)}(t,x)].\end{aligned}$$

Then, for closed-loop switched system (6), we have the following result.

Lemma 2: If there exist functions α_1, $\alpha_2\in\mathcal{K}_\infty$, class $C^{2,1}$ Lyapunov function $V_{\bar{\sigma}(t)}(x,t)$ and some positive constants λ_s, λ_a and $\mu\geq 1$, such that

$$\alpha_1(|x|)\leq V_{\bar{\sigma}(t)}(x,t)\leq\alpha_2(|x|); \tag{7}$$

and for any $l\in\mathbb{N}$,

$$LV_{\bar{\sigma}(t)}(x,t)\leq\begin{cases}-\lambda_s V_{\bar{\sigma}(t)}(x,t), & t\in T_s(t_l,t_{l+1}),\\ \lambda_a V_{\bar{\sigma}(t)}(x,t), & t\in T_a(t_l,t_{l+1}),\end{cases}\tag{8}$$

hold almost surely. For any $r\in\mathbb{N}_+$,

$$\mathbb{E}\{V_{\bar{\sigma}(\bar{t}_r)}(x(\bar{t}_r),\bar{t}_r)\} \le \mu\mathbb{E}\{V_{\bar{\sigma}(\bar{t}_{r-1})}(x(\bar{t}_r),\bar{t}_r)\}. \tag{9}$$

Further, if there also exist some nonnegative constants $\rho \ge 0$ such that

$$\rho < \frac{\lambda_s}{\lambda_s + \lambda_a}, \tag{10}$$

and for any $t \ge t_0$,

$$T_a(t-t_0) \le \rho(t-t_0). \tag{11}$$

Then, system (6) is GASiP for all $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$.

Proof: Denote $W_{\bar{\sigma}(t)}(x,t)$ by $e^{\lambda_s t}V_{\bar{\sigma}(t)}(x,t)$. Then, in each time interval $[t_l, t_{l+1})$, according to inequality (8), we can obtain that

$$LW_{\bar{\sigma}(t)}(x,t) \le \begin{cases} 0, & t \in T_s(t_l,t_{l+1}); \\ (\lambda_s+\lambda_a)W_{\bar{\sigma}(t)}(x,t), & t \in T_a(t_l,t_{l+1}). \end{cases}$$

According to (9) and $\mathbb{E}\{LW_{\bar{\sigma}(t)}(x,t)\} = \frac{\mathrm{d}}{\mathrm{d}t}\mathbb{E}\{W_{\bar{\sigma}(t)}(x,t)\}$, when $t \in T_a(t_l,t_{l+1})$, $\mathbb{E}\{W_{\bar{\sigma}}(x(t),t)\} \le \mathbb{E}\{W_{\bar{\sigma}}(x(t_l),t_l)\}e^{(\lambda_s+\lambda_a)(t-t_l)}$; when $t \in T_s(t_l,t_{l+1}) \cup [t_0,t_1)$, $\mathbb{E}\{W_{\bar{\sigma}(t)}(x(t),t)\} \le \mathbb{E}\{W_{\bar{\sigma}(t)}(x(t_l+T_a(t_{l+1}-t_l)), t_l+T_a(t_{l+1}-t_l))\}$. Then, for $t \in [t_l,t_{l+1})$, $l \in \mathbb{N}$, it holds that $\mathbb{E}\{W_{\bar{\sigma}(t)}(x(t),t)\} \le \mu\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_{2l})}(x(t_l),t_l)\}e^{(\lambda_s+\lambda_a)T_a(t-t_l)}$. Thus,

$$\begin{aligned}
&\mathbb{E}\{W_{\bar{\sigma}(t)}(x(t),t)\} \le \mu\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_{2l})}(x(t_l),t_l)\}e^{(\lambda_s+\lambda_a)T_a(t-t_l)} \\
&\le \mu^2\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_{2l-1})}x(t_l),t_l)\}e^{(\lambda_s+\lambda_a)T_a(t-t_l)} \\
&\le \mu^3\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_{2(l-1)})}(x(t_{l-1}),t_{l-1})\}e^{(\lambda_s+\lambda_a)T_a(t-t_{l-1})} \\
&\le \mu^5\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_{2(l-2)})}(x(t_{l-2}),t_{l-2})\}e^{(\lambda_s+\lambda_a)T_a(t-t_{l-2})} \\
&\le \cdots \\
&\le \mu^{2N_{\bar{\sigma}}(t,t_0)}\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_0)}(x(t_0),t_0)\}e^{(\lambda_s+\lambda_a)T_a(t-t_0)} \\
&\le \mu^{2N_0+\frac{2\bar{d}}{\tau^*}}\mathbb{E}\{W_{\bar{\sigma}(t_0)}(x(t_0),t_0)\}e^{[(\lambda_s+\lambda_a)\rho+\frac{\ln(\mu)}{\tau^*}](t-t_0)}
\end{aligned}$$

for any $t \in [t_l,t_{l+1})$, $l \in \mathbb{N}$. Then,

$$\begin{aligned}
&\mathbb{E}\{V_{\bar{\sigma}(t)}(x(t),t)\} \\
&\le \mu^{2N_0+\frac{2\bar{d}}{\tau^*}}V_{\bar{\sigma}(t_0)}(x(t_0),t_0)e^{[(\lambda_s+\lambda_a)\rho+\frac{\ln(\mu)}{\tau^*}-\lambda_s](t-t_0)} \\
&\triangleq \bar{\beta}(V_{\bar{\sigma}(t_0)}(x(t_0),t_0),t-t_0).
\end{aligned}$$

When $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$, it's easy to verify that $\bar{\beta} \in \mathcal{KL}$. For any $\varepsilon > 0$, take $\tilde{\beta} = \dfrac{\bar{\beta}}{\varepsilon} \in \mathcal{KL}$. By Chebyshev's inequality, we have

$$\begin{aligned}
&\mathbb{P}\{V_{\bar{\sigma}(t)}(x(t),t) \ge \tilde{\beta}(V_{\bar{\sigma}(t_0)}(x_0,t_0),t-t_0)\} \\
&\le \frac{\mathbb{E}\{V_{\bar{\sigma}(t)}(x(t),t)\}}{\tilde{\beta}(V_{\bar{\sigma}(t_0)}(x(t_0),t_0),t-t_0)} < \varepsilon, \forall t \in [t_l,t_{l+1}).
\end{aligned}$$

Let $\beta(r,s) = \alpha_1^{-1}\circ\tilde{\beta}(\bar{\alpha}_2(r),s)$. β is a $\mathcal{KL}$ function if the average dwell time is satisfied. Thus, we have

$$\mathbb{P}\{|x(t)| < \beta(|x_0|,t-t_0)\} \ge 1-\varepsilon, \forall t \ge t_0.$$

This completes the proof.

Remark 2: The conditions (10) and (11) in Lemma 2 implies that the considered system can be stable provided that the mismatched period is sufficiently small. The condition (8) indicates that the switched control systems can start from unstable systems. Change the condition (11) into the one that $\forall t \ge t_0 : T_a(t-t_0) \le \tau_0 + \rho(t-t_0)$, where τ_0 can be interpreted as an initial offset which allows us to start with a subsystem with mismatched controller. Clearly, we have $0 \le \tau_0 \le t_1 - t_0$. Then, do th-e similar analysis, we can also get the conclusion.

The following result can be obtained directly from the proof of Lemma 2.

Corollary 1: System (6) is pth moment exponentially stable for all $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$, if α_1 and α_2 in Lemma 2 are such that $\alpha_1(s) = c_1 s^p$ and $\alpha_2(s) = c_2 s^p$ where c_1 and c_2 are positive constants.

The following lemma is useful for the stability of retarded system (3) under asynchronous switching.

Lemma 3: For any $C^{2,1}$ function $V_{\bar{\sigma}}(x,t)$, let $U(t) = V_{\bar{\sigma}}(x,t)$ for $t \ge t_0$. Then $\mathbb{E}\{U(t)\}$ is continuous.

Proof: Based on $It\hat{o}$'s formula, we have

$$\begin{aligned}
V_{\bar{\sigma}(t)}(x,t) &= V_{\bar{\sigma}(t_0)}(x(t_0),t_0) + \int_{t_0}^{t}\mathcal{L}V_{\bar{\sigma}(s)}(x_s,s)\mathrm{d}s \\
&\quad + \int_{t_0}^{t}\frac{\partial V_{\bar{\sigma}(s)}(x(s),s)}{\partial x}\bar{g}_{\bar{\sigma}(s)}(s,x(s),x_s)\mathrm{d}w(s).
\end{aligned} \tag{12}$$

Since $\xi \in C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, we can find an integer k_0 such that $\|\xi\| < k_0$ a.s. Then, for each integer $k > k_0$, define a sequence of stopping time ρ_k as $\rho_k = \inf\{t \ge t_0 : |x(t)| > k, \forall k > k_0\}$. Clearly, $\rho_k \to \infty$ as $k \to \infty$. If t is replaced by $\tau_k = t \wedge \rho_k$ in (12), then the stochastic integral (second integral) in (12) defines a martingale (with k fixed and t varying), not just a local martingale. Thus, $\mathbb{E}\{V_{\bar{\sigma}(\tau_k)}(x(\tau_k),\tau_k)\} = \mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t_0),t_0)\} + \mathbb{E}\{\int_{t_0}^{\tau_k}\mathcal{L}V_{\bar{\sigma}(s)}(x_s,s)\mathrm{d}s\}$. Letting $k \to \infty$ and using Fubini's theorem incorporated with Fatou's lemma yields

$$\begin{aligned}
\mathbb{E}\{U(t)\} &= \mathbb{E}\{U(t_0)\} + \mathbb{E}\left\{\int_{t_0}^{t}\mathcal{L}V_{\bar{\sigma}(s)}(x_s,s)\mathrm{d}s\right\} \\
&= \mathbb{E}\{U(t_0)\} + \int_{t_0}^{t}\mathbb{E}\left\{\mathcal{L}V_{\bar{\sigma}(s)}(x_s,s)\right\}\mathrm{d}s,
\end{aligned}$$

for all $t \geq t_0$, which implies $\mathbb{E}\{U(t)\}$ is continuous.

Theorem 1: Suppose there exist functions $\alpha_1 \in \mathcal{K}_\infty$, $\alpha_2 \in \mathcal{VK}_\infty$, class $C^{2,1}$ Lyapunov function $V_{\bar{\sigma}(t)}(x,t)$ and some constants $\lambda_s > 0$, $\lambda_a > 0$, $\mu \geq 1$, $\rho \geq 0$ and $q > 1$, such that inequalities (7) and (9)-(11) hold, and moreover, for any $l \in \mathbb{N}$,

$$\mathbb{E}\{\mathcal{L}V_{\bar{\sigma}(t)}(\varphi(\theta),t)\} < \begin{cases} -\lambda_s \mathbb{E}\{V_{\bar{\sigma}(t)}(\varphi(0),t)\}, & t \in T_s(t_l,t_{l+1}); \\ \lambda_a \mathbb{E}\{V_{\bar{\sigma}(t)}(\varphi(0),t)\}, & t \in T_a(t_l,t_{l+1}), \end{cases} \tag{13}$$

provided those $\varphi \in L^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$ satisfying that

$$\min_{i,j\in\mathcal{S}} \mathbb{E}\{V_{ij}(\varphi(\theta),t+\theta)\} \leq q\mathbb{E}\{V_{\bar{\sigma}(t)}(\varphi(0),t)\}, \tag{14}$$

for any $\theta \in [-\tau,0]$. Further, if we also have

$$e^{(\lambda_s(1-\rho)-\lambda_a\rho)\tau} \leq q\ . \tag{15}$$

Then, system (3) is GASiP for all $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$.

Proof: According to (7) and Jensen's inequality, we obtain that

$$\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t_0+\theta),t_0+\theta)\} \leq \mathbb{E}\{\alpha_2(|x(t_0+\theta)|)\} \leq \alpha_2(\mathbb{E}\{\|\xi\|\}) \leq Me^{[\lambda_s(\rho-1)+\lambda_a\rho]\theta},$$

for any $\theta \in [-\tau,0]$, where $M \triangleq \alpha_2(\mathbb{E}\{\|\xi\|\})$. Then, we have $\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t),t)\} \leq Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t-t_0)}$, $t \in [t_0-\tau, t_0]$. Now, for any $t \in [t_0,t_1)$, we prove that

$$\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t),t)\} \leq Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t-t_0)}. \tag{16}$$

Suppose there exists some $t \in (t_0,t_1)$ such that $\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t),t)\} > Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t-t_0)}$. Let $t^* = \inf\{t \in (t_0,t_1) : \mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t),t)\} > Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t-t_0)}\}$. Considering the continuity of $V_{\bar{\sigma}(t_0)}$ and $x(t)$ on $[t_0,t_1)$, without loss of generality, we have $t^* \in (t_0,t_1)$ and $\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\} = Me^{(\lambda_s(\rho-1)+\lambda_a)\rho(t^*-t_0)}$. Further, there exists a sequence $\{\tilde{t}_n\}$ ($\tilde{t}_n \in (t^*,t_1)$, for any $n \in \mathbb{N}_+$) with $\lim_{n\to\infty} \tilde{t}_n = t^*$, such that $\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(\tilde{t}_n),\tilde{t}_n)\} > Me^{[\lambda_s(\rho-1)+\lambda_a\rho](\tilde{t}_n-t_0)}$. From the definition of t^*, we have

$$\begin{aligned}\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*+\theta),t^*+\theta)\} &\leq \mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\} \\ &\leq e^{(\lambda_s(\rho-1)+\lambda_a\rho)\theta}\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\} \\ &\leq q\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\},\end{aligned}$$

and further

$$\min_{i,j\in\mathcal{S}} \mathbb{E}\{V_{ij}(x(t+\theta),t+\theta)\} \leq q\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\},$$

for any $\theta \in [-\tau,0]$. On the other hand, from Lemma 3, we have $D^+\mathbb{E}\{V_{\bar{\sigma}}(x,t)\} = \mathbb{E}\{\mathcal{L}V_{\bar{\sigma}}(x_t,t)\}$, where

$$D^+\mathbb{E}\{V_{\bar{\sigma}(t)}(x,t)\} = \limsup_{h\to 0^+} \frac{\mathbb{E}\{V_{\bar{\sigma}(t+h)}(x(t+h),t+h)\} - \mathbb{E}\{V_{\bar{\sigma}(t)}(x,t)\}}{h}.$$

From inequality (13), we can obtain

$$D^+\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\} < -\lambda_s\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\}. \tag{17}$$

for any $t \in [t_0,t_1)$. Clearly, there exists a positive constants $h > 0$, which is small enough, such that (17) holds on $[t^*,t^*+h]$. Then,

$$\mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*+h),t^*+h)\} \leq \mathbb{E}\{V_{\bar{\sigma}(t_0)}(x(t^*),t^*)\}e^{-\lambda_s h} \leq Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t^*-t_0)},$$

which is contradiction proves (16).

Considering the continuity, we further get that (16) holds on all interval $[t_0,t_1]$. According to condition (9), when $t = \bar{t}_1 = t_1$, we have

$$\mathbb{E}\{V_{\bar{\sigma}(\bar{t}_1)}(x(t_1),t_1)\} \leq \mu Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t_1-t_0)} \tag{18}$$

Let $W_{\bar{\sigma}(t)}(t) = e^{\lambda_s t}V_{\bar{\sigma}(t)}(x(t),t)$, then, $\mathbb{E}\{W_{\bar{\sigma}(\bar{t}_1)}(t_1)\} \leq \mu\, \mathbb{E}\{W_{\bar{\sigma}(t_0)}(t_1)\}$. For any $l \in \mathbb{N}$, in time interval $[t_l,t_{l+1})$, we also have

$$\mathcal{L}W_{\bar{\sigma}(t)}(t) < \begin{cases} 0, & t \in T_s(t_l,t_{l+1}); \\ (\lambda_s+\lambda_a)W_{\bar{\sigma}(t)}(t), & t \in T_a(t_l,t_{l+1}). \end{cases}$$

Similarly, from Lemma 2 and Remark 2, we can obtain

$$\begin{aligned}\mathbb{E}\{W_{\bar{\sigma}(t)}(t)\} &\leq \mu^{2N_{\bar{\sigma}(t)}(t,t_0)}\mathbb{E}\{W_{\bar{\sigma}(t_0)}(t_1)\}e^{(\lambda_s+\lambda_a)T_a(t-t_1)} \\ &= \mu^{2N_{\bar{\sigma}(t)}(t,t_0)}\mathbb{E}\{W_{\bar{\sigma}(t_0)}(t_1)\}e^{(\lambda_s+\lambda_a)T_a(t-t_0)},\end{aligned}$$

which means

$$\begin{aligned}&\mathbb{E}\{V_{\bar{\sigma}(t)}(x(t),t)\} \\ &\leq \mu^{2N_{\bar{\sigma}(t)}(t,t_0)}Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t_1-t_0)}e^{(\lambda_s+\lambda_a)T_a(t-t_0)}e^{-\lambda_s(t-t_1)} \\ &\leq \mu^{2N_0+\frac{2\bar{d}}{\tau^*}}e^{\rho(\lambda_s+\lambda_a)(t_1-t_0)}Me^{[(\lambda_s+\lambda_a)\rho-\lambda_s+\frac{\ln(\mu)}{\tau^*}](t-t_0)}.\end{aligned}$$

This completes the proof by Lemma 2.

Theorem 2: Let $\varsigma = \sup_{l\in\mathbb{N}_+}\{t_l - t_{l-1}\} < \infty$. Suppose there exist functions $\alpha_1 \in \mathcal{K}_\infty$, $\alpha_2 \in \mathcal{VK}_\infty$, class $C^{2,1}$ Lyapunov function $V_{\bar{\sigma}(t)}(x,t)$ and some constants $\lambda_s > 0$, $\lambda_a > 0$, $\mu \geq 1$, and $q > 1$, such that inequalities (7), (9), (13) and (14) hold. Further, if there also exists nonnegative constant ρ, such that for any $t \geq \bar{\tau} \geq t_0$,

$$\rho < \frac{\lambda_s}{\lambda_s+\lambda_a}, \tag{19}$$

$$T_a(t-\bar{\tau}) \leq \rho(t-\bar{\tau}). \tag{20}$$

Moreover, condition (15) is also satisfied. Then, system (3) is GASiP for all $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$.

Proof: Following the proof of Theorem 1, we have

$$\mathbb{E}\{V_{\bar\sigma(t_0)}(x(t),t)\} \le Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t-t_0)}, \quad \forall t\in[t_0,t_1],$$

and

$$\mathbb{E}\{V_{\bar\sigma(\bar t_1)}(x(t_1),t_1)\} \le \mu\mathbb{E}\{V_{\bar\sigma(t_0)}(x(t_1),t_1)\} < \mu Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t_1-t_0)}.$$

Divide the time interval into $T_a(t_1,t_2)$ and $T_s(t_1,t_2)$. From condition (13), if (14) holds, we have

$$D^+\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\} < \begin{cases} -\lambda_s\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\}, & t\in T_s(t_1,t_2);\\ \lambda_a\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\}, & t\in T_a(t_1,t_2).\end{cases}$$

Let $W_{\bar\sigma}(t)=e^{\lambda_s t}V_{\bar\sigma}(x,t)$. By the continuity of $V_{\bar\sigma}(x,t)$ on any interval $[t_{l-1},t_l)$, $l\in\mathbb{N}_+$, it's easy to verify that $\mathbb{E}\{W_{\bar\sigma(t)}(t)\}$ is continuous on $[t_{l-1},t_l)$. Moreover, $\mathcal{L}W_{\bar\sigma(t)}(t)=\lambda_s W_{\bar\sigma(t)}(t)+e^{\lambda_s t}\mathcal{L}V_{\bar\sigma(t)}(x_t,t)$. Then

$$D^+\mathbb{E}\{W_{\bar\sigma(t)}(t)\} < \begin{cases} 0, & t\in T_s(t_1,t_2);\\ (\lambda_s+\lambda_a)\mathbb{E}\{W_{\bar\sigma(t)}(t)\}, & t\in T_a(t_1,t_2).\end{cases}$$

Similar to Theorem 1, for any $t\in[t_1,t_2)$, we have

$$\mathbb{E}\{W_{\bar\sigma(t)}(t)\} \le \mu\mathbb{E}\{W_{\bar\sigma(\bar t_1)}(t_1)\}e^{(\lambda_s+\lambda_a)T_a(t_2-t_1)},$$

i.e.,

$$\begin{aligned}&\mathbb{E}\{V_{\bar\sigma(t)}(x,t)\}\\ &\le \mu\mathbb{E}\{V_{\bar\sigma(\bar t_1)}(x(t_1),t_1)\}e^{-\lambda_s(t-t_1)+(\lambda_s+\lambda_a)T_a(t_2-t_1)}\\ &= \mu^2Me^{-\lambda_s(t-t_0)}e^{(\lambda_s+\lambda_a)(T_a(t_2-t_1)+\rho(t_1-t_0))}.\end{aligned}$$

By the continuity of $V_{\bar\sigma(t)}(x,t)$ and x, we have

$$\begin{aligned}&\mathbb{E}\{V_{\bar\sigma(t_1)}(x(t_2),t_2)\}\\ &\le \mu^2Me^{-\lambda_s(t-t_0)}e^{(\lambda_s+\lambda_a)(T_a(t_2-t_1)+\rho(t_1-t_0))}.\end{aligned}$$

Now, suppose that for some $l\ge 2$, we have

$$\begin{aligned}&\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\}\\ &\le \mu^{2l}Me^{-\lambda_s(t-t_0)}e^{(\lambda_s+\lambda_a)(\sum_{i=1}^{l}T_a(t_{i+1}-t_i)+\rho(t_1-t_0))},\end{aligned}$$

for any $t\in[t_l,t_{l+1})$. If for any $t\in[t_{l+1},t_{l+2}]$, we have

$$\begin{aligned}\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\} &\le \mu^{2l+2}Me^{-\lambda_s(t-t_0)}\\ &\quad\times e^{(\lambda_s+\lambda_a)(\sum_{i=1}^{l+1}T_a(t_{i+1}-t_i)+\rho(t_1-t_0))},\end{aligned} \tag{21}$$

then by mathematical induction, the above assumption holds. Actually, by the above induction, (21) holds clearly. Thus, for all $t\ge t_0$,

$$\begin{aligned}\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\} &\le \mu^{2N_{\bar\sigma}(t,t_0)}Me^{(\lambda_s+\lambda_a)\sum_{i=1}^{N_{\bar\sigma}(t,t_0)}T_a(t_{i+1}-t_i)}\\ &\quad\times e^{(\lambda_s+\lambda_a)\rho(t_1-t_0)}e^{-\lambda_s(t-t_0)}.\end{aligned} \tag{22}$$

Combining (20) and (22), it follows that

$$\begin{aligned}&\mathbb{E}\{V_{\bar\sigma(t)}(x(t),t)\}\\ &\le \mu^{2N_{\bar\sigma}(t,t_0)}Me^{-\lambda_s(t-t_0)}e^{(\lambda_s+\lambda_a)\rho(t-t_0)}e^{(\lambda_s+\lambda_a)\rho(t_{N_{\bar\sigma}(t,t_0)+1}-t)}\\ &\le \mu^{2N_{\bar\sigma}(t,t_0)}Me^{[\lambda_s(\rho-1)+\lambda_a\rho](t-t_0)}e^{(\lambda_s+\lambda_a)\rho\varsigma}\\ &\le \mu^{2N_0+\frac{2\bar d}{\tau^*}}e^{(\lambda_s+\lambda_a)\rho\varsigma}Me^{[\lambda_s(\rho-1)+\lambda_a\rho+\frac{\ln(\mu)}{\tau^*}](t-t_0)}\\ &\triangleq \tilde\beta(\mathbb{E}\{\|\xi\|\},t-t_0).\end{aligned}$$

If $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$, then $\tilde\beta(\cdot,\cdot)\in\mathcal{KL}$. The proof is completed similarly to Lemma 2.

Remark 3: λ_a and λ_s are referred to the instability margin and the stability margin, respectively. From (15), it is seen that for any fixed q, τ and stability margin λ_s, a large instability margin λ_a can be compensated by a small $0<\rho<1$. That is to say, the switched system is stable provided that the proportion of mismatched time is small enough.

Remark 4: Condition (20) requires this upper bound to hold uniformly over any interval $[\bar\tau,t)$ with arbitrary starting point $\bar\tau\le t$, which offers a simple strategy to design the switching signal. Since all detection delays are bounded, we can design the switching signal with sufficiently large average dwell-time directly.

Similar to Corollary 1, the following useful corollary is obtained.

Corollary 2: System (3) is pth moment exponentially stable for all $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho)-\lambda_a\rho}$, if α_1 and α_2 in Theorem 1 or in Theorem 2 are such that $\alpha_1(s)=c_1s^p$ and $\alpha_2(s)=c_2s^p$ where c_1 and c_2 are positive constants.

Remark 5: In reference [24], by using the Razumikhin method and the average dwell time approach, the criterion of pth moment exponentially stability for a class of switched stochastic nonlinear systems is developed. However, the focus of our work is on stability analysis under asynchronous switching, which is very different from [24], and this is also the major contribution of our work. In fact, if we let $\sigma'(t)\equiv\sigma(t)$, i.e., we consider synchronous switching, then the closed-loop dynamic in (3) is the same as (2.1) in [24]. In this case, $\rho\equiv 0$ in Corollary 2. Then, the result in Corollary 2 is the same the result in [24].

Remark 6: In this paper, we consider only the detection delay, assumed to satisfy some conditions, i.e., $d_\sigma(t)\le\bar d\le\inf_{l\in\mathbb{N}}\{t_{l+1}-t_l\}$. However, if there exists a detection error, then the assumption may not hold. (The same problem also appears in the results in reference [41-43], etc.) This difficulty leaves for our future endeavor.

4. APPLICATION AND EXAMPLE

We apply the general Razumikhin-type Corollary 2 to deal with the pth moment exponentially stability for a special type of switched stochastic nonlinear delay feedback systems, where the controller is designed with both state and switching delays.

Consider the following switched stochastic nonlinear control system

$$\begin{cases} dx = F_\sigma(t,x,y_1,u(t))dt + G_\sigma(t,x,y_1,u(t))dw, \\ u(t) = H_{\sigma'(t)}(t,y_2), \\ x(s) = \phi(s), \sigma(s) = \sigma(t_0) = i_0, t_0 - \tau \le s \le t_0, \end{cases} \tag{23}$$

on $t \ge t_0$, where $y_1(t) = x(t-d_1(t))$, $y_2(t) = x(t-d_2(t))$. $0 \le \max\{d_1(t), d_2(t)\} \le \tau$. Assume $F_i : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}^n$, $G_i : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}^{n\times m}$ and $H_i : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^l$ are continuous with $F_i(t,0,0,0) \equiv 0$ and $G_i(t,0,0,0) \equiv 0$ for all $i \in \mathcal{S}$, and moreover system (23) has the unique solution. For convenience, we transfer the system (23) into the following type, i.e.,

$$dx = \bar{F}_{\bar{\sigma}(t)}(t,x,y_1,y_2)dt + \bar{G}_{\bar{\sigma}(t)}(t,x,y_1,y_2)dw. \tag{24}$$

For any given $V_{\bar{\sigma}(t)} \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+; \mathbb{R}_+)$, the diffusion operator $\mathcal{L}V_{\bar{\sigma}(t)}$ in (5) becomes from $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S} \times \mathcal{S}$ to $\mathbb{R}$ by [18]:

$$\begin{aligned} &\mathcal{L}V_{\bar{\sigma}(t)}(x,y_1,y_2,t) \\ &= \frac{\partial V_{\bar{\sigma}(t)}(x,t)}{\partial t} + \frac{\partial V_{\bar{\sigma}(t)}(x,t)}{\partial x}\bar{F}_{\bar{\sigma}(t)}(t,x,y_1,y_2) \\ &+ \frac{1}{2} trace[\bar{G}_{\bar{\sigma}(t)}(t,x,y_1,y_2)\frac{\partial^2 V_{\bar{\sigma}(t)}(x,t)}{\partial x^2}\bar{G}_{\bar{\sigma}(t)}(t,x,y_1,y_2)]. \end{aligned}$$

Using Corollary 2, the following result is obtained.

Corollary 3: Let $\varsigma = \sup_{l\in\mathbb{N}_+}\{t_l - t_{l-1}\} < \infty$. For each $i_l \in \mathcal{S}$, $l \in \mathbb{N}$, suppose there exist a class $C^{2,1}$ Lyapunov function $V_{\bar{\sigma}(t)}(x(t),t)$ and some positive constants c_1, c_2, λ_{si_l}, λ_{i_l}, λ_{ai_l}, $\mu \ge 1$ and $q > 1$, such that

$$c_1 |x(t)|^p \le V_{\bar{\sigma}(t)}(x(t),t) \le c_2 |x(t)|^p, \tag{25}$$

and when $t \in T_s(t_l, t_{l+1})$ with $T_s(t_0,t_1) = [t_0, t_1)$,

$$\begin{aligned} &\mathcal{L}V_{\bar{\sigma}(t)}(x,y_1,y_2,t) \\ &< -\lambda_{si_l}V_{\bar{\sigma}(t)}(x,t) + \sum_{k=1}^{2}\lambda_{i_l k}\min_{i,j\in\mathcal{S}} V_{ij}(y_k, t-d_k(t)); \end{aligned} \tag{26}$$

when $t \in T_a(t_l, t_{l+1})$ with $T_a(t_0,t_1) = \varnothing$,

$$\begin{aligned} &\mathcal{L}V_{\bar{\sigma}(t)}(x,y_1,y_2,t) \\ &< \lambda_{ai_l}V_{\bar{\sigma}(t)}(x,t) + \sum_{k=1}^{2}\lambda_{i_l k}\min_{i,j\in\mathcal{S}} V_{ij}(y_k, t-d_k(t)), \end{aligned} \tag{27}$$

provided those $\varphi \in L^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$ satisfying that

$$\min_{i,j\in\mathcal{S}} \mathbb{E}\{V_{ij}(\varphi(\theta), t+\theta)\} \le q\mathbb{E}\{V_{\bar{\sigma}(t)}(\varphi(0),t)\} \tag{28}$$

for any $\theta \in [-\tau, 0]$. And for all $r \in \mathbb{N}_+$, we have

$$\mathbb{E}\{V_{\bar{\sigma}(\bar{t}_r)}(x(\bar{t}_r),\bar{t}_r)\} \le \mu\mathbb{E}\{V_{\bar{\sigma}(\bar{t}_{r-1})}(x(\bar{t}_r),\bar{t}_r)\}. \tag{29}$$

Let $\lambda_k = \max_{i_l\in\mathcal{S}}\{\lambda_{i_l k}\}$, $\lambda_s = \min_{i_l\in\mathcal{S}}\{\lambda_{si_l}\} - \sum_{k=1}^{2}\lambda_k q > 0$ and $\lambda_a = \max_{i_l\in\mathcal{S}}\{\lambda_{ai_l}\} + \sum_{k=1}^{2}\lambda_k q$. If there also exist some nonnegative constant ρ, such that (19), (20) and (15) hold. Then, system (24) is pth moment exponentially stable for all $\tau^* > \dfrac{\ln(\mu)}{\lambda_s(1-\rho) - \lambda_a\rho}$.

Proof: Taking the expectation on the both sides of (26) and (27), by Fatou's lemma and from (28), we have

$$\begin{aligned} &\mathbb{E}\{\mathcal{L}V_{\bar{\sigma}(t)}(x(t),y_1(t),y_2(t),t)\} \\ &< \begin{cases} -\lambda_s\mathbb{E}\{V_{\bar{\sigma}(t)}(x(t),t)\}, t \in T_s(t_l,t_{l+1}); \\ \lambda_a\mathbb{E}\{V_{\bar{\sigma}(t)}(x(t),t)\}, t \in T_a(t_l,t_{l+1}). \end{cases} \end{aligned}$$

For any $t \ge t_0$ and $\varphi \in L^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$, define

$$\bar{f}_{\bar{\sigma}(t)}(t,\varphi(0),\varphi) = \bar{F}_{\bar{\sigma}(t)}(t,\varphi(0),\varphi(-d_1(t)),\varphi(-d_2(t))),$$

and

$$\bar{g}_{\bar{\sigma}(t)}(t,\varphi(0),\varphi) = \bar{G}_{\bar{\sigma}(t)}(t,\varphi(0),\varphi(-d_1(t)),\varphi(-d_2(t))).$$

Thus, all the conditions in Corollary 2 are satisfied. This completes the proof.

The following example is considered to demonstrate the effectiveness of the proposed method.

Example 1: Consider the following switched stochastic nonlinear systems.

$$dx = [A_{\sigma(t)}x + B_{\sigma(t)}u + f_{\sigma(t)}(t,x)]dt + C_{\sigma(t)}x dw,$$

where $f_i : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is an unknown nonlinear functions satisfying $|f_i(t,x(t))| \le \|U_i\|_2 |x(t)|$, for any $i \in \mathcal{S}$, where $\|\cdot\|_2$ denotes the 2-norm of the matrix. Set $u(t) = K_{\sigma'(t)}y(t) = K_{\sigma'(t)}x(t-d(t))$, then

$$\begin{aligned} dx &= [A_{\sigma(t)}x + B_{\sigma(t)}K_{\sigma'(t)}y + f_{\sigma(t)}(t,x)]dt \\ &+ C_{\sigma(t)}x dw. \end{aligned} \tag{30}$$

Let $\bar{\sigma}(t) = (\sigma(t), \sigma'(t)) \in \mathcal{S}\times\mathcal{S}$. For system (30), take $V_{\bar{\sigma}(t)}(x) = x^T P_{\bar{\sigma}(t)}x$. Following reference [46], the conclusion that $HFE + E^T F^T H^T \le \varepsilon HH^T + \varepsilon^{-1}E^T E$, holds for any $\varepsilon > 0$, when $F^T F \le I$, where I is an identity matrix with appropriate dimension. Then

$$\begin{aligned} \mathcal{L}V_{ii}(x) &\le x^T[A_i^T P_{ii} + P_{ii}A_i + C_i^T P_{ii}C_i + \varepsilon_1 P_{ii} + \varepsilon_2 P_{ii}]x \\ &+ \varepsilon_1^{-1}y^T K_i^T B_i^T P_{ii}B_i K_i y + \varepsilon_2^{-1}f_i^T(t,x)P_{ii}f_i(t,x), \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}V_{ij}(x) &\le x^T[A_i^T P_{ij} + P_{ij}A_i + C_i^T P_{ij}C_i + \varepsilon_3 P_{ij} + \varepsilon_4 P_{ij}]x \\ &+ \varepsilon_3^{-1}y^T K_j^T B_i^T P_{ij}B_i K_j y + \varepsilon_4^{-1}f_i^T(t,x)P_{ij}f_i(t,x), \end{aligned}$$

for any $i, j \in \mathcal{S}$ and $j \neq i$. Let $X_i = P_{ii}^{-1}$ and $K_{ii} = X_i K_i^T$. If there exist positive constants β_1 and β_2 such that

$$X_i > \beta_1^{-1} I, \quad P_{ij} < \beta_2 I. \tag{31}$$

Then,

$$\mathcal{L}V_{ii}(x) \leq x^T \left[A_i^T P_{ii} + P_{ii} A_i + C_i^T P_{ii} C_i + \varepsilon_1 P_{ii} + \varepsilon_2 P_{ii} + \varepsilon_2^{-1} \beta_1 U_i^T U_i \right] x + \varepsilon_1^{-1} y^T K_i^T B_i^T P_{ii} B_i K_i y,$$

and

$$\mathcal{L}V_{ij}(x) \leq x^T \left[A_i^T P_{ij} + P_{ij} A_i + C_i^T P_{ij} C_i + \varepsilon_3 P_{ij} + \varepsilon_4 P_{ij} + \varepsilon_4^{-1} \beta_2 U_i^T U_i \right] x + \varepsilon_3^{-1} y^T K_j^T B_i^T P_{ij} B_i K_j y.$$

According to Corollary 3, if there exist positive constants λ_{si}, λ_{ai} and λ_i such that

$$x^T \left[A_i^T P_{ii} + P_{ii} A_i + C_i^T P_{ii} C_i + \varepsilon_1 P_{ii} + \varepsilon_2 P_{ii} + \varepsilon_2^{-1} \beta_1 U_i^T U_i \right] x + \varepsilon_1^{-1} y^T K_i^T B_i^T P_{ii} B_i K_i y < -\lambda_{si} x^T P_{ii} x + \lambda_i y^T P_{ii} y,$$

and

$$x^T \left[A_i^T P_{ij} + P_{ij} A_i + C_i^T P_{ij} C_i + \varepsilon_3 P_{ij} + \varepsilon_4 P_{ij} + \varepsilon_4^{-1} \beta_2 U_i^T U_i \right] x + \varepsilon_3^{-1} y^T K_j^T B_i^T P_{ij} B_i K_j y < \lambda_{ai} x^T P_{ij} x + \lambda_i y^T P_{ij} y,$$

which means

$$\Omega = diag\{\Omega_{11}, \Omega_{22}\} < 0, \tag{32}$$

$$\Sigma = diag\{\Sigma_{11}, \Sigma_{22}\} < 0, \tag{33}$$

where $\Omega_{11} = A_i^T P_{ii} + P_{ii} A_i + C_i^T P_{ii} C_i + \varepsilon_1 P_{ii} + \varepsilon_2 P_{ii} + \varepsilon_2^{-1} \times \beta_1 U_i^T U_i + \lambda_{si} P_{ii}$, $\Omega_{22} = -\lambda_i P_{ii} + \varepsilon_1^{-1} K_i^T B_i^T P_{ii} B_i K_i$, $\Sigma_{11} = A_i^T P_{ij} + P_{ij} A_i + C_i^T P_{ij} C_i + \varepsilon_3 P_{ij} + \varepsilon_4 P_{ij} + \varepsilon_4^{-1} \beta_2 U_i^T U_i - \lambda_{ai} P_{ij}$, $\Sigma_{22} = -\lambda_i P_{ij} + \varepsilon_3^{-1} K_j^T B_i^T P_{ij} B_i K_j$, $\varepsilon_k > 0$, $k = 1,2,3,4$. Using $diag\{P_{ii}^{-1}, P_{ii}^{-1}\}$ to pre- and post- multiply the left terms of matrix inequality (32); and using Schur's complement lemma, then (32) is equivalent to

$$\begin{bmatrix} \bar{\Omega}_{11} & X_i U_i & X_i C_i^T & 0 & 0 \\ * & -\varepsilon_2 \beta_1^{-1} I & 0 & 0 & 0 \\ * & * & -X_i & 0 & 0 \\ * & * & * & -\lambda_i X_i & K_{ii} B_i^T \\ * & * & * & * & -\varepsilon_1 X_i \end{bmatrix} < 0, \tag{34}$$

where $\bar{\Omega}_{11} = X_i A_i^T + A_i X_i + \varepsilon_1 X_i + \varepsilon_2 X_i + \lambda_{si} X_i$. Moreover, if there also exist $q > 1$ and $\mu \geq 1$, such that, P_{ii}, P_{ij}, λ_{si}, λ_{ai}, λ_i, q and μ satisfy the corresponding conditions in Corollary 3, then system (30) is 2nd moment exponentially stable.

Specify system (30) as follows

$$A_1 = \begin{bmatrix} -5 & -2 \\ 5 & -3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -4 & 0 \\ 1 & -5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0 \\ -0.3 & 0.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.3 & -0.2 \\ 0 & 0.5 \end{bmatrix},$$

$$f_1(t,x) = \begin{bmatrix} 0.5\cos(t) & 0.1\sin(|x|) \\ 0 & -0.1\sin(t) \end{bmatrix} x,$$

$$U_1 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & -0.1 \end{bmatrix},$$

$$f_2(t,x) = \begin{bmatrix} 0.1\cos(t)\sin(|x|) & 0 \\ 0 & 0.5\sin(t) \end{bmatrix} x,$$

$$U_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

$d(t) = 0.8\cos(t)$. Take $\lambda_{s1} = 2.8$, $\lambda_{s2} = 3$, $\lambda_{a1} = 0.03$, $\lambda_{a2} = 0.01$, $\lambda_1 = 0.07$, $\lambda_2 = 0.11$, $\beta_1 = 1$, $\varepsilon_1 = 1.2$, $\varepsilon_2 = 1.5$, $\varepsilon_3 = 3$ and $\varepsilon_4 = 4$. Then, using the LMI toolbox in the MATLAB, we get

$$P_{11} = \begin{bmatrix} 0.5762 & 0.0354 \\ 0.0354 & 0.2707 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 0.2647 & 0.0399 \\ 0.0212 & 0.2050 \end{bmatrix},$$

$$P_{12} = \begin{bmatrix} 189.9085 & 44.5369 \\ 44.5369 & 87.1685 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.4129 & 0.1255 \\ 0.1191 & 0.1606 \end{bmatrix},$$

$$P_{22} = \begin{bmatrix} 0.3792 & -0.0493 \\ -0.0493 & 0.4606 \end{bmatrix},$$

$$P_{21} = \begin{bmatrix} 128.9302 & -6.1472 \\ -6.1472 & 96.2041 \end{bmatrix},$$

and $\mu = 1.7737$, $\beta_2 = 268.2155$. Take $q = 3$, then $\lambda = 0.11$, $\lambda_s = 2.47$, and $\lambda_a = 0.36$. Set $\tau = 0.8$, $\rho = \frac{1}{2}\frac{\lambda_s}{\lambda_a + \lambda_s} = 0.4364$, then $3 = q > e^{(\lambda_s(1-\rho) - \lambda_a \rho)\tau} = 2.6859$. Then, according to the above analysis, system (30) is 2nd moment exponentially stable for all $\tau^* > 0.4640$s. The simulation results are shown in Figs. 1, 2, and 3. Figs. 1 and 2 show the Brownian motion $w(t)$ and the switching signal in system (30), respectively. More-over, the detection delay in Fig. 2 satisfies the conditions in Corollary 3. Finally, Fig. 3 shows the trajectory of $x(t)$ under the initial data $x_0 = (\pm 8, \mp 6)$. Under the asynchronous switching signal $\sigma'(t)$ in Fig. 2, the state $x(t)$ will converge to zero.

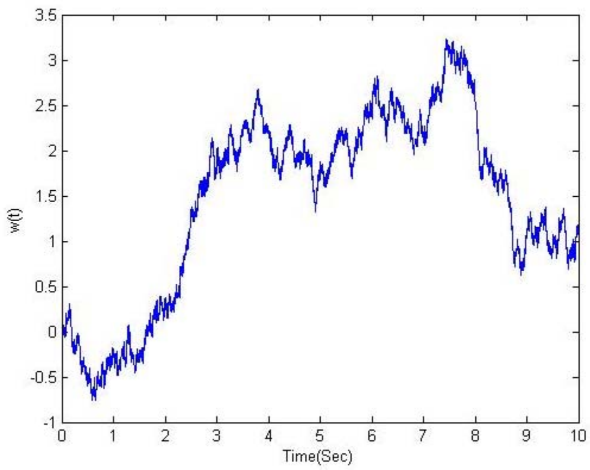

Fig. 1. Response curve of Brownian motion $w(t)$.

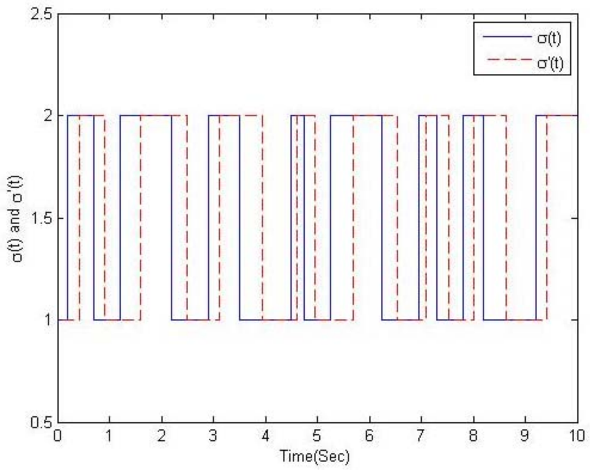

Fig. 2. Switching signal $\sigma(t)$ and the detected $\sigma'(t)$.

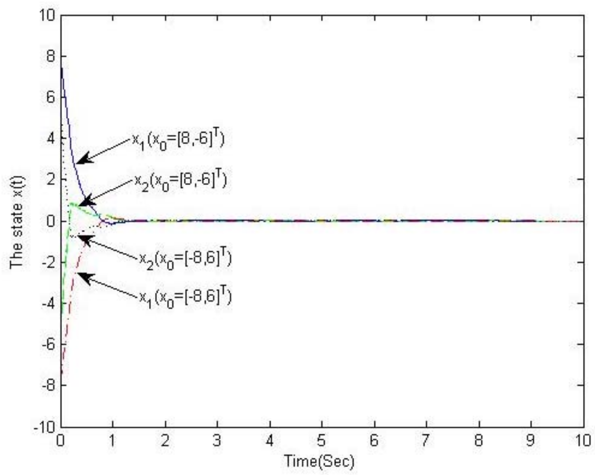

Fig. 3. Response curve of $x(t)$.

5. CONCLUSION

The stability of a class of stochastic nonlinear retarded systems under asynchronous switching is investigated. Based on the average dwell time approach, the corresponding Razumikhin-type stability criteria on globally asymptotically stable as well as pth moment exponentially stable are given. It is shown that the switched system can be stable when the mismatched interval is small enough while the average dwell time is large enough. Finally, we apply the results to a class of stochastic nonlinear delay systems where the design of controller is considered with both state and switching delays and meaningful results are obtained, which are illustrated by a numerical example.

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